# HL Paper 2

The population of mosquitoes in a specific area around a lake is controlled by pesticide. The rate of decrease of the number of mosquitoes is proportional to the number of mosquitoes at any time *t*. Given that the population decreases from 500 000 to 400 000 in a five year period, find the time it takes in years for the population of mosquitoes to decrease by half.

# Markscheme

Let the number of mosquitoes be *y*.

$\frac{\mathrm{d}y}{\mathrm{d}t} = -ky$ ***M1***

$\int \frac{1}{y}\mathrm{d}y = \int -k\mathrm{d}t$ ***M1***

$\ln y = -kt + c$ ***A1***

$y = \mathrm{e}^{-kt+c}$

$y = A\mathrm{e}^{-kt}$

when $t = 0,\ y = 500\,000 \Rightarrow A = 500\,000$ ***A1***

$y = 500\,000\mathrm{e}^{-kt}$

when $t = 5,\ y = 400\,000$

$400\,000 = 500\,000\mathrm{e}^{-5k}$ ***M1***

$\frac{4}{5} = \mathrm{e}^{-5k}$

$-5k = \ln\frac{4}{5}$

$k = -\frac{1}{5}\ln\frac{4}{5} \quad (= 0.0446)$ ***A1***

$250\,000 = 500\,000\mathrm{e}^{-kt}$ ***M1***

$\frac{1}{2} = \mathrm{e}^{-kt}$

$\ln\frac{1}{2} = -kt$

$t = \frac{5}{\ln\frac{4}{5}}\ln\frac{1}{2} = 15.5 \text{ years}$ ***A1***

***[8 marks]***

# Examiners report

Some candidates assumed that the decrease in population size was exponential / geometric and were therefore unable to gain the first 4 marks. Apart from this, reasonably good attempts were made by many candidates.

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The acceleration in $\mathrm{ms}^{-2}$ of a particle moving in a straight line at time $t$ seconds, $t \geqslant 0$ , is given by the formula $a = -\frac{1}{2}v$. When $t = 0$ , the velocity is $40\ \mathrm{ms}^{-1}$ .

Find an expression for $v$ in terms of $t$ .

# Markscheme

$\frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{1}{2}v$ ***A1***

$\int \frac{\mathrm{d}v}{v} = \int -\frac{1}{2}\mathrm{d}t$ ***(A1)***

$\ln v = -\frac{1}{2}t + c$ ***(A1)***

$v = \mathrm{e}^{-\frac{1}{2}t+c} \ \left(= A\mathrm{e}^{-\frac{1}{2}t}\right)$ ***(A1)***

$t = 0, v = 40$, so $A = 40$ ***M1***

$v = 40\mathrm{e}^{-\frac{1}{2}t}$ (or equivalent) ***A1***

***[6 marks]***

# Examiners report

This was a poorly answered question which linked the topic of kinematics with that of first order differential equations. Many candidates seemed unaware that the acceleration is the time derivative of the velocity. This was often followed by a failure to recognize a separable differential equation and/or integration with respect to the wrong variable.

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An open glass is created by rotating the curve $y = x^2$ , defined in the domain $x \in [0, 10]$, $2\pi$ radians about the $y$-axis. Units on the coordinate axes are defined to be in centimetres.

a. When the glass contains water to a height $h$ cm, find the volume $V$ of water in terms of $h$ . [3]

b. If the water in the glass evaporates at the rate of 3 $\text{cm}^3$ per hour for each $\text{cm}^2$ of exposed surface area of the water, show that, [6]

$\frac{\mathrm{d}V}{\mathrm{d}t} = -3\sqrt{2\pi V}$ , where $t$ is measured in hours.

c. If the glass is filled completely, how long will it take for all the water to evaporate? [7]

# Markscheme

a. volume $= \pi\int_0^h x^2\mathrm{d}y$ ***(M1)***

$\pi\int_0^h y\mathrm{d}y$ ***M1***

$\pi\left[\frac{y^2}{2}\right]_0^h = \frac{\pi h^2}{2}$ ***A1***

***[3 marks]***

b. $\frac{\mathrm{d}V}{\mathrm{d}t} = -3\times$ surface area ***A1***

surface area $= \pi x^2$ ***(M1)***

$= \pi h$ ***A1***

$V = \frac{\pi h^2}{2} \Rightarrow h\sqrt{\frac{2V}{\pi}}$ ***M1A1***

$\frac{\mathrm{d}V}{\mathrm{d}t} = -3\pi\sqrt{\frac{2V}{\pi}}$ ***A1***

$\frac{\mathrm{d}V}{\mathrm{d}t} = -3\sqrt{2\pi V}$ ***AG***

**Note:** Assuming that $\frac{\mathrm{d}h}{\mathrm{d}t} = -3$ without justification gains no marks.

***[6 marks]***

c. $V_0 = 5000\pi\ (= 15700\ \mathrm{cm}^3)$ ***A1***

$\frac{\mathrm{d}V}{\mathrm{d}t} = -3\sqrt{2\pi V}$

attempting to separate variables ***M1***

**EITHER**

$\int \frac{\mathrm{d}V}{\sqrt{V}} = -3\sqrt{2\pi}\int \mathrm{d}t$ ***A1***

$2\sqrt{V} = -3\sqrt{2\pi}t + c$ ***A1***

$c = 2\sqrt{5000\pi}$ ***A1***

$V = 0$ ***M1***

$\Rightarrow t = \frac{2}{3}\sqrt{\frac{5000\pi}{2\pi}} = 33\frac{1}{3}$ hours ***A1***

**OR**

$\int_{5000\pi}^{0} \frac{\mathrm{d}V}{\sqrt{V}} = -3\sqrt{2\pi}\int_0^T \mathrm{d}t$ ***M1A1A1***

**Note:** Award ***M1*** for attempt to use definite integrals, ***A1*** for correct limits and ***A1*** for correct integrands.

$\left[2\sqrt{V}\right]_{5000\pi}^{0} = 3\sqrt{2\pi}T$ ***A1***

$T = \frac{2}{3}\sqrt{\frac{5000\pi}{2\pi}} = 33\frac{1}{3}$ hours ***A1***

***[7 marks]***

# Examiners report

a. This question was found to be challenging by many candidates and there were very few completely correct solutions. Many candidates did not seem able to find the volume of revolution when taken about the *y*-axis in (a). Candidates did not always recognize that part (b) did not involve related rates. Those candidates who attempted the question made some progress by separating the variables and integrating in (c) but very few were able to identify successfully the values necessary to find the correct answer.

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Consider the differential equation $y\frac{dy}{dx} = \cos 2x$.

a. (i) Show that the function $y = \cos x + \sin x$ satisfies the differential equation. [10]

(ii) Find the general solution of the differential equation. Express your solution in the form $y = f(x)$, involving a constant of integration.

(iii) For which value of the constant of integration does your solution coincide with the function given in part (i)?

b. A different solution of the differential equation, satisfying $y = 2$ when $x = \frac{\pi}{4}$, defines a curve $C$. [12]

(i) Determine the equation of $C$ in the form $y = g(x)$, and state the range of the function $g$.

A region $R$ in the $xy$ plane is bounded by $C$, the $x$-axis and the vertical lines $x = 0$ and $x = \frac{\pi}{2}$.

(ii) Find the area of $R$.

(iii) Find the volume generated when that part of $R$ above the line $y = 1$ is rotated about the $x$-axis through $2\pi$ radians.

# Markscheme

a. (i) **METHOD 1**

$\frac{dy}{dx} = -\sin x + \cos x$ ***A1***

$y\frac{dy}{dx} = (\cos x + \sin x)(-\sin x + \cos x)$ ***M1***

$= \cos^2 x - \sin^2 x$ ***A1***

$= \cos 2x$ ***AG***

**METHOD 2**

$y^2 = (\sin x + \cos x)^2$ ***A1***

$2y\frac{dy}{dx} = 2(\cos x + \sin x)(\cos x - \sin x)$ ***M1***

$y\frac{dy}{dx} = \cos^2 x - \sin^2 x$ ***A1***

$= \cos 2x$ ***AG***

(ii) attempting to separate variables $\int y\,dy = \int \cos 2x\,dx$ ***M1***

$\frac{1}{2}y^2 = \frac{1}{2}\sin 2x + C$ ***A1A1***

**Note:** Award ***A1*** for a correct LHS and ***A1*** for a correct RHS.

$y = \pm(\sin 2x + A)^{\frac{1}{2}}$ ***A1***

(iii) $\sin 2x + A \equiv (\cos x + \sin x)^2$ ***(M1)***

$(\cos x+\sin x)^2=\cos^2 x+2\sin x\cos x+\sin^2 x$

use of $\sin 2x\equiv 2\sin x\cos x$. ***(M1)***

$A=1$ ***A1***

***[10 marks]***

b. (i) substituting $x=\frac{\pi}{4}$ and $y=2$ into $y=(\sin 2x+A)^{\frac{1}{2}}$ ***M1***

so $g(x)=(\sin 2x+3)^{\frac{1}{2}}$. ***A1***

range $g$ is $\left[\sqrt{2},\ 2\right]$ ***A1A1A1***

**Note:** Accept [1.41, 2]. Award ***A1*** for each correct endpoint and ***A1*** for the correct closed interval.

(ii) $\int_0^{\frac{\pi}{2}}(\sin 2x+3)^{\frac{1}{2}}\mathrm{d}x$ ***(M1)(A1)***

= 2.99 ***A1***

(iii) $\pi\int_0^{\frac{\pi}{2}}(\sin 2x+3)\mathrm{d}x-\pi(1)\left(\frac{\pi}{2}\right)$ (or equivalent) ***(M1)(A1)(A1)***

**Note:** Award ***(M1)(A1)(A1)*** for $\pi\int_0^{\frac{\pi}{2}}(\sin 2x+2)\mathrm{d}x$

$=17.946-4.935\ (=\frac{\pi}{2}(3\pi+2)-\pi\left(\frac{\pi}{2}\right))$ ***A1***

**Note:** Award ***A1*** for $\pi(\pi+1)$.

***[12 marks]***

# Examiners report

a. Part (a) was not well done and was often difficult to mark. In part (a) (i), a large number of candidates did not know how to verify a solution, $y(x)$, to the given differential equation. Instead, many candidates attempted to solve the differential equation. In part (a) (ii), a large number of candidates began solving the differential equation by correctly separating the variables but then either neglected to add a constant of integration or added one as an afterthought. Many simple algebraic and basic integral calculus errors were seen. In part (a) (iii), many candidates did not realize that the solution given in part (a) (i) and the general solution found in part (a) (ii) were to be equated. Those that did know to equate these two solutions, were able to square both solution forms and correctly use the trigonometric identity $\sin 2x=2\sin x\cos x$. Many of these candidates however started with incorrect solution(s).

b. In part (b), a large number of candidates knew how to find a required area and a required volume of solid of revolution using integral calculus. Many candidates, however, used incorrect expressions obtained in part (a). In part (b) (ii), a number of candidates either neglected to state 'π' or attempted to calculate the volume of a solid of revolution of 'radius' $f(x)-g(x)$.

The acceleration of a car is $\frac{1}{40}(60 - v)$ ms$^{-2}$, when its velocity is $v$ ms$^{-2}$. Given the car starts from rest, find the velocity of the car after 30 seconds.

# Markscheme

**METHOD 1**

$\frac{dv}{dt} = \frac{1}{40}(60 - v)$ ***(M1)***

attempting to separate variables $\int \frac{dv}{60-v} = \int \frac{dt}{40}$ ***M1***

$-\ln(60 - v) = \frac{t}{40} + c$ ***A1***

$c = -\ln 60$ (or equivalent) ***A1***

attempting to solve for $v$ when $t = 30$ ***(M1)***

$v = 60 - 60e^{-\frac{3}{4}}$

$v = 31.7\ (\text{ms}^{-1})$ ***A1***

**METHOD 2**

$\frac{dv}{dt} = \frac{1}{40}(60 - v)$ ***(M1)***

$\frac{dt}{dv} = \frac{40}{60-v}$ (or equivalent) ***M1***

$\int_0^{v_f} \frac{40}{60-v} dv = 30$ where $v_f$ is the velocity of the car after 30 seconds. ***A1A1***

attempting to solve $\int_0^{v_f} \frac{40}{60-v} dv = 30$ for $v_f$ ***(M1)***

$v = 31.7\ (\text{ms}^{-1})$ ***A1***

***[6 marks]***

# Examiners report

Most candidates experienced difficulties with this question. A large number of candidates did not attempt to separate the variables and instead either attempted to integrate with respect to $v$ or employed constant acceleration formulae. Candidates that did separate the variables and attempted to integrate both sides either made a sign error, omitted the constant of integration or found an incorrect value for this constant. Almost all candidates were not aware that this question could be solved readily on a GDC.

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(a) Solve the differential equation $\frac{\cos^2 x}{e^y} - e^{e^y}\frac{dy}{dx} = 0$, given that $y = 0$ when $x = \pi$.

(b) Find the value of y when $x = \frac{\pi}{2}$.

# Markscheme

(a) rearrange $\frac{\cos^2 x}{e^y} - e^{e^y}\frac{dy}{dx} = 0$ to obtain $\cos^2 x dx = e^y e^{e^y} dy$ ***(M1)***

as $\int \cos^2 x dx = \int \frac{1+\cos(2x)}{2} dx = \frac{1}{2}x + \frac{1}{4}\sin(2x) + C_1$ ***M1A1***

and $\int e^y e^{e^y} dy = e^{e^y} + C_2$ ***A1***

**Note:** The above two integrations are independent and should not be penalized for missing.

a general solution of $\frac{\cos^2 x}{e^y} - e^{e^y}\frac{dy}{dx} = 0$ is $\frac{1}{2}x + \frac{1}{4}\sin(2x) - e^{e^y} = C$ ***A1***

given that $y = 0$ when $x = \pi$, $C = \frac{\pi}{2} + \frac{1}{4}\sin(2\pi) - e^{e^0} = \frac{\pi}{2} - e$, (or $-1.15$) ***(M1)***

so, the required solution is defined by the equation

$\frac{1}{2}x + \frac{1}{4}\sin(2x) - e^{e^y} = \frac{\pi}{2} - e$ or $y = \ln\left(\ln\left(\frac{1}{2}x + \frac{1}{4}\sin(2x) + e^{e^y} - \frac{\pi}{2}\right)\right)$ ***A1*** ***N0***

(or equivalent)

(b) for $x = \frac{\pi}{2}$, $y = \ln\left(\ln\left(e - \frac{\pi}{4}\right)\right)$ (or $-0.417$ ) ***A1***

***[8 marks]***

# Examiners report

This was a more difficult question and it was apparent that students did find it so. For those that managed to rearrange the equation to separate the variables, few could manage to successfully integrate both sides. The unfamiliarity of $e^{e^y}$ seemed to disturb some students.

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A. Prove by mathematical induction that, for $n \in \mathbb{Z}^+$, [8]

$$1 + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)^3 + \ldots + n\left(\frac{1}{2}\right)^{n-1} = 4 - \frac{n+2}{2^{n-1}}.$$

B. (a) Using integration by parts, show that $\int e^{2x}\sin x\,dx = \frac{1}{5}e^{2x}(2\sin x - \cos x) + C$ . [17]

(b) Solve the differential equation $\frac{dy}{dx} = \sqrt{1 - y^2}e^{2x}\sin x$, given that $y = 0$ when $x = 0$,

writing your answer in the form $y = f(x)$ .

(c) (i) Sketch the graph of $y = f(x)$ , found in part (b), for $0 \leqslant x \leqslant 1.5$ .

Determine the coordinates of the point P, the first positive intercept on the $x$-axis, and mark it on your sketch.

(ii) The region bounded by the graph of $y = f(x)$ and the $x$-axis, between the origin and P, is rotated 360° about the $x$-axis to form a solid of revolution.

Calculate the volume of this solid.

# Markscheme

A. prove that $1 + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)^3 + \ldots + n\left(\frac{1}{2}\right)^{n-1} = 4 - \frac{n+2}{2^{n-1}}$

for $n = 1$

$\text{LHS} = 1,\ \text{RHS} = 4 - \frac{1+2}{2^0} = 4 - 3 = 1$

so true for $n = 1$ ***R1***

assume true for $n = k$ ***M1***

so $1 + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)^3 + \ldots + k\left(\frac{1}{2}\right)^{k-1} = 4 - \frac{k+2}{2^{k-1}}$

now for $n = k + 1$

LHS: $1 + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)^3 + \ldots + k\left(\frac{1}{2}\right)^{k-1} + (k+1)\left(\frac{1}{2}\right)^k$ ***A1***

$= 4 - \frac{k+2}{2^{k-1}} + (k+1)\left(\frac{1}{2}\right)^k$ ***M1A1***

$= 4 - \frac{2(k+2)}{2^k} + \frac{k+1}{2^k}$ (or equivalent) ***A1***

$= 4 - \frac{(k+1)+2}{2^{(k+1)-1}}$ (accept $4 - \frac{k+3}{2^k}$) ***A1***

Therefore if it is true for $n = k$ it is true for $n = k + 1$. It has been shown to be true for $n = 1$ so it is true for all $n$ $(\in \mathbb{Z}^+)$. ***R1***

**Note:** To obtain the final ***R*** mark, a reasonable attempt at induction must have been made.

***[8 marks]***

B. (a)

**METHOD 1**

$\int e^{2x} \sin x \mathrm{d}x = -\cos x e^{2x} + \int 2e^{2x} \cos x \mathrm{d}x$ ***M1A1A1***

$= -\cos x e^{2x} + 2e^{2x} \sin x - \int 4e^{2x} \sin x \mathrm{d}x$ ***A1A1***

$5 \int e^{2x} \sin x \mathrm{d}x = -\cos x e^{2x} + 2e^{2x} \sin x$ ***M1***

$\int e^{2x} \sin x \mathrm{d}x = \frac{1}{5} e^{2x}(2 \sin x - \cos x) + C$ ***AG***

**METHOD 2**

$\int \sin x e^{2x} \mathrm{d}x = \frac{\sin x e^{2x}}{2} - \int \cos x \frac{e^{2x}}{2} \mathrm{d}x$ ***M1A1A1***

$= \frac{\sin x e^{2x}}{2} - \cos x \frac{e^{2x}}{4} - \int \sin x \frac{e^{2x}}{4} \mathrm{d}x$ ***A1A1***

$\frac{5}{4} \int e^{2x} \sin x \mathrm{d}x = \frac{e^{2x} \sin x}{2} - \frac{\cos x e^{2x}}{4}$ ***M1***

$\int e^{2x} \sin x \mathrm{d}x = \frac{1}{5} e^{2x}(2 \sin x - \cos x) + C$ ***AG***

***[6 marks]***

(b)

$\int \frac{\mathrm{d}y}{\sqrt{1-y^2}} = \int e^{2x} \sin x \mathrm{d}x$ ***M1A1***

$\arcsin y = \frac{1}{5} e^{2x}(2 \sin x - \cos x)(+C)$ ***A1***

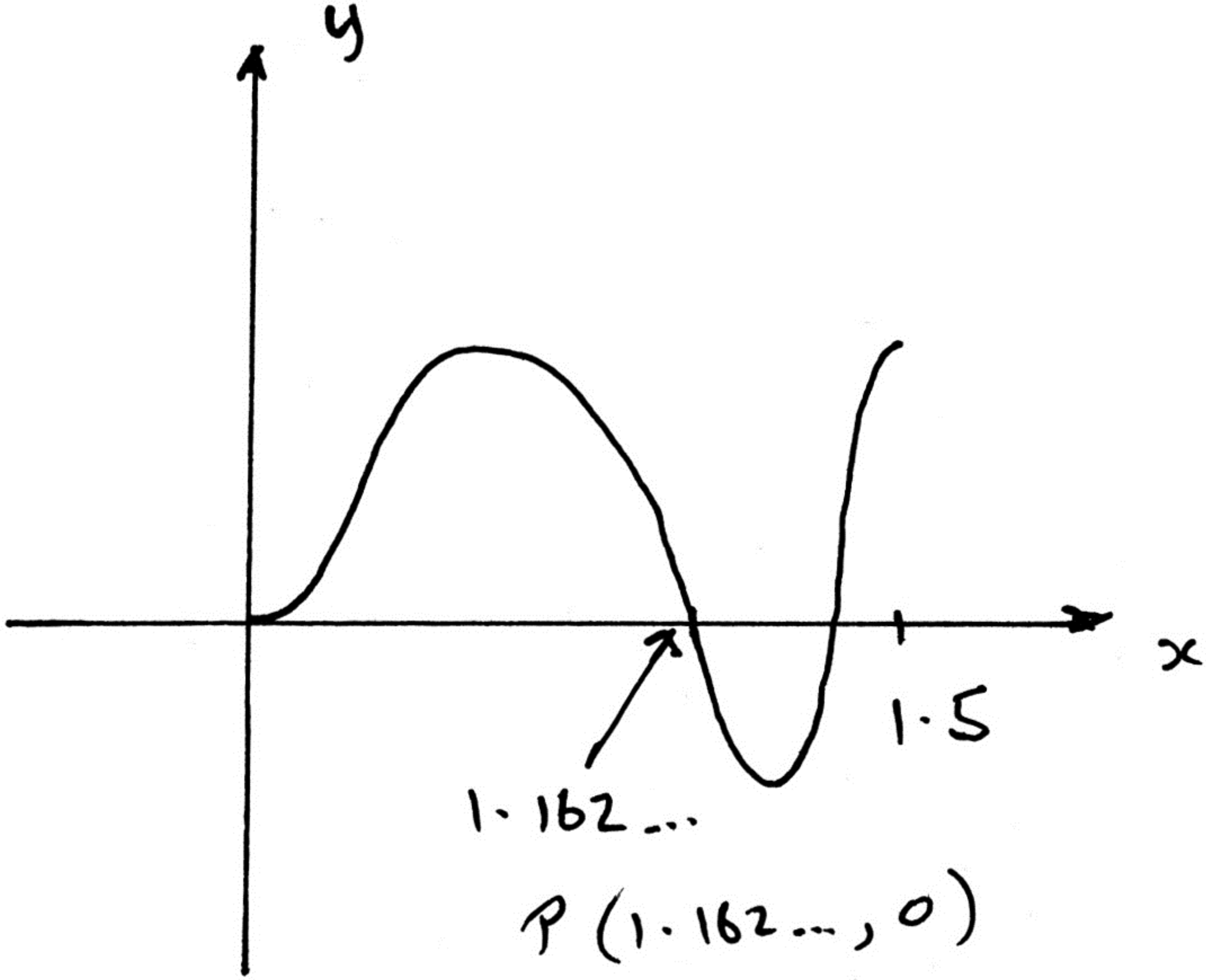
when $x = 0,\ y = 0 \Rightarrow C = \frac{1}{5}$ ***M1***

$y = \sin\left(\frac{1}{5} e^{2x}(2 \sin x - \cos x) + \frac{1}{5}\right)$ ***A1***

***[5 marks]***

(c)

(i)




*A1*

P is (1.16, 0) ***A1***

**Note:** Award ***A1*** for 1.16 seen anywhere, ***A1*** for complete sketch.

**Note:** Allow FT on their answer from (b)

(ii) $V = \int_0^{1.162...} \pi y^2 \mathrm{d}x$ ***M1A1***

$= 1.05$ ***A2***

**Note:** Allow FT on their answers from (b) and (c)(i).

***[6 marks]***

# Examiners report

A. Part A: Given that this question is at the easier end of the ‘proof by induction’ spectrum, it was disappointing that so many candidates failed to score full marks. The $n = 1$ case was generally well done. The whole point of the method is that it involves logic, so ‘let n = k’ or ‘put n = k’, instead of ‘assume ... to be true for n = k’, gains no marks. The algebraic steps need to be more convincing than some candidates were able to show. It is astonishing that the R1 mark for the final statement was so often not awarded.

B. Part B: Part (a) was often well done, although some faltered after the first integration. Part (b) was also generally well done, although there were some errors with the constant of integration. In (c) the graph was often attempted, but errors in (b) usually led to manifestly incorrect plots. Many attempted the volume of integration and some obtained the correct value.

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A particle moves in a straight line with velocity $v$ metres per second. At any time $t$ seconds, $0 \leqslant t < \frac{3\pi}{4}$, the velocity is given by the differential equation $\frac{\mathrm{d}v}{\mathrm{d}t} + v^2 + 1 = 0$ .
It is also given that $v = 1$ when $t = 0$ .

a. Find an expression for $v$ in terms of $t$ . [7]

b. Sketch the graph of $v$ against $t$ , clearly showing the coordinates of any intercepts, and the equations of any asymptotes. [3]

c. (i) Write down the time $T$ at which the velocity is zero. [3]
(ii) Find the distance travelled in the interval $[0, T]$ .

d. Find an expression for $s$ , the displacement, in terms of $t$ , given that $s = 0$ when $t = 0$ . [5]

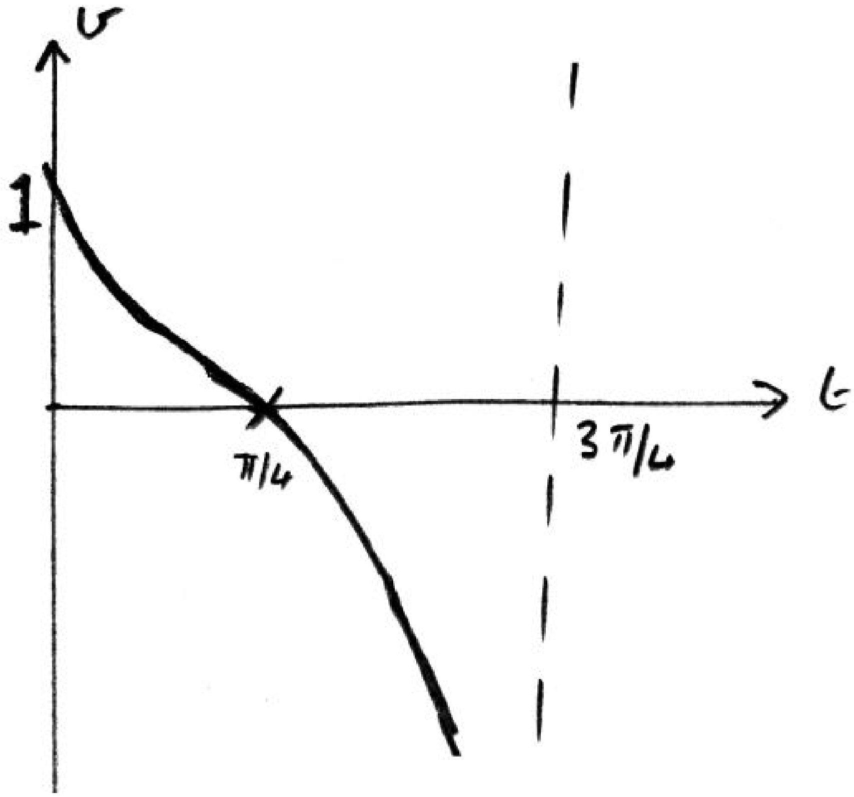
e. Hence, or otherwise, show that $s = \frac{1}{2}\ln \frac{2}{1+v^2}$. [4]

# Markscheme

a. $\frac{\mathrm{d}v}{\mathrm{d}t} = -v^2 - 1$
attempt to separate the variables ***M1***
$\int \frac{1}{1+v^2}\mathrm{d}v = \int -1\mathrm{d}t$ ***A1***
$\arctan v = -t + k$ ***A1A1***
**Note:** Do not penalize the lack of constant at this stage.

when $t = 0$, $v = 1$ ***M1***
$\Rightarrow k = \arctan 1 = \left(\frac{\pi}{4}\right) = (45^\circ)$ ***A1***
$\Rightarrow v = \tan\left(\frac{\pi}{4} - t\right)$ ***A1***
***[7 marks]***

b.




*A1A1A1*

**Note:** Award ***A1*** for general shape,
***A1*** for asymptote,
***A1*** for correct *t* and *v* intercept.

**Note:** Do not penalise if a larger domain is used.

***[3 marks]***

c. (i) $T = \frac{\pi}{4}$ ***A1***

(ii) area under curve $= \int_0^{\frac{\pi}{4}} \tan\left(\frac{\pi}{4} - t\right) \mathrm{d}t$ ***(M1)***

$= 0.347 \left(= \frac{1}{2}\ln 2\right)$ ***A1***

***[3 marks]***

d. $v = \tan\left(\frac{\pi}{4} - t\right)$

$s = \int \tan\left(\frac{\pi}{4} - t\right) \mathrm{d}t$ ***M1***

$\int \frac{\sin\left(\frac{\pi}{4} - t\right)}{\cos\left(\frac{\pi}{4} - t\right)} \mathrm{d}t$ ***(M1)***

$= \ln\cos\left(\frac{\pi}{4} - t\right) + k$ ***A1***

when $t = 0,\ s = 0$

$k = -\ln\cos\frac{\pi}{4}$ ***A1***

$s = \ln\cos\left(\frac{\pi}{4} - t\right) - \ln\cos\frac{\pi}{4} \left(= \ln\left[\sqrt{2}\cos\left(\frac{\pi}{4} - t\right)\right]\right)$ ***A1***

***[5 marks]***

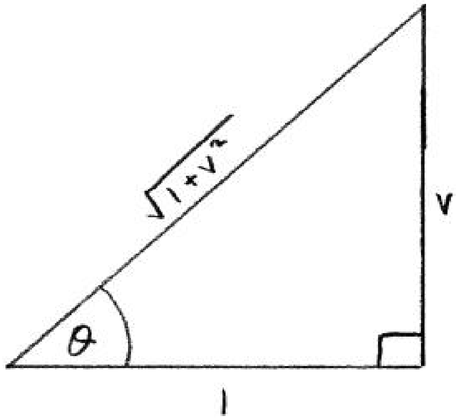
e. **METHOD 1**

$\frac{\pi}{4} - t = \arctan v$ ***M1***

$t = \frac{\pi}{4} - \arctan v$

$s = \ln\left[\sqrt{2}\cos\left(\frac{\pi}{4} - \frac{\pi}{4} + \arctan v\right)\right]$

$s = \ln\left[\sqrt{2}\cos(\arctan v)\right]$ ***M1A1***




$s = \ln\left[\sqrt{2}\cos\left(\arccos\frac{1}{\sqrt{1+v^2}}\right)\right]$ ***A1***

$= \ln\frac{\sqrt{2}}{\sqrt{1+v^2}}$

$= \frac{1}{2}\ln\frac{2}{1+v^2}$ ***AG***

**METHOD 2**

$s = \ln\cos\left(\frac{\pi}{4} - t\right) - \ln\cos\frac{\pi}{4}$

$= -\ln\sec\left(\frac{\pi}{4} - t\right) - \ln\cos\frac{\pi}{4}$ ***M1***

$= -\ln\sqrt{1 + \tan^2\left(\frac{\pi}{4} - t\right)} - \ln\cos\frac{\pi}{4}$ ***M1***

$= -\ln\sqrt{1 + v^2} - \ln\cos\frac{\pi}{4}$ ***A1***

$= \ln\frac{1}{\sqrt{1+v^2}} + \ln\sqrt{2}$ ***A1***

$= \frac{1}{2}\ln\frac{2}{1+v^2}$ ***AG***

**METHOD 3**

$v\frac{dv}{ds} = -v^2 - 1$ ***M1***

$\int\frac{v}{v^2+1}dv = -\int 1ds$ ***M1***

$\frac{1}{2}\ln(v^2 + 1) = -s + k$ ***A1***

when $s = 0\,,\ t = 0 \Rightarrow v = 1$

$\Rightarrow k = \frac{1}{2}\ln 2$ ***A1***

$\Rightarrow s = \frac{1}{2}\ln\frac{2}{1+v^2}$ ***AG***

***[4 marks]***

# Examiners report

a. This proved to be the most challenging question in section B with only a very small number of candidates producing fully correct answers.

Many candidates did not realise that part (a) was a differential equation that needed to be solved using a method of separating the variables.

Without this, further progress with the question was difficult. For those who did succeed in part (a), parts (b) and (c) were relatively well done.

For the minority of candidates who attempted parts (d) and (e) only the best recognised the correct methods.

b. This proved to be the most challenging question in section B with only a very small number of candidates producing fully correct answers.

Many candidates did not realise that part (a) was a differential equation that needed to be solved using a method of separating the variables.

Without this, further progress with the question was difficult. For those who did succeed in part (a), parts (b) and (c) were relatively well done.

For the minority of candidates who attempted parts (d) and (e) only the best recognised the correct methods.

c. This proved to be the most challenging question in section B with only a very small number of candidates producing fully correct answers. Many candidates did not realise that part (a) was a differential equation that needed to be solved using a method of separating the variables. Without this, further progress with the question was difficult. For those who did succeed in part (a), parts (b) and (c) were relatively well done. For the minority of candidates who attempted parts (d) and (e) only the best recognised the correct methods.

d. This proved to be the most challenging question in section B with only a very small number of candidates producing fully correct answers. Many candidates did not realise that part (a) was a differential equation that needed to be solved using a method of separating the variables. Without this, further progress with the question was difficult. For those who did succeed in part (a), parts (b) and (c) were relatively well done. For the minority of candidates who attempted parts (d) and (e) only the best recognised the correct methods.

e. This proved to be the most challenging question in section B with only a very small number of candidates producing fully correct answers. Many candidates did not realise that part (a) was a differential equation that needed to be solved using a method of separating the variables. Without this, further progress with the question was difficult. For those who did succeed in part (a), parts (b) and (c) were relatively well done. For the minority of candidates who attempted parts (d) and (e) only the best recognised the correct methods.